Heterogeneous Differential Privacy

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The Theory and Practice of Differential Privacy
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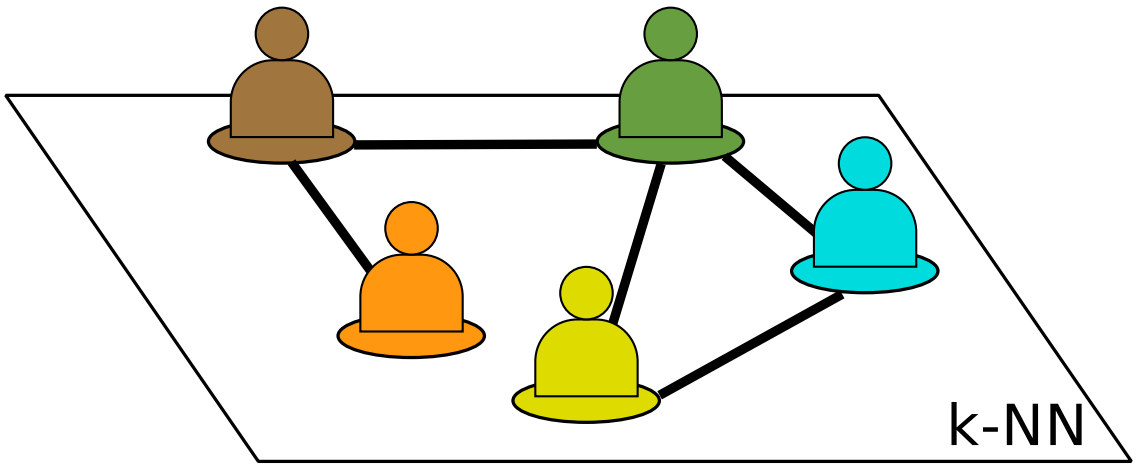
Context

- Personalization through user-based collaborative filtering
  - Users who share similar interests like similar items
- P2P system:
  - Users host their own data
  - They meet at random and find those who share their interests
    - By computing a similarity metric (probably based on the inner product)
  - They construct a k-Nearest Neighbor overlay network[1]



[1] The Gossple Anonymous Social Network [BFGKL 2010]

System Model

- There are n items (URLs, videos, images, news pieces,...)
- Each user is associated with a private profile representing his interest with respect to these items (like/dislike, etc...)
- Profiles are binary vectors $\in \{0,1\}^n$
  - 1 means like
  - 0 means either dislike, or has never been rated

Example:

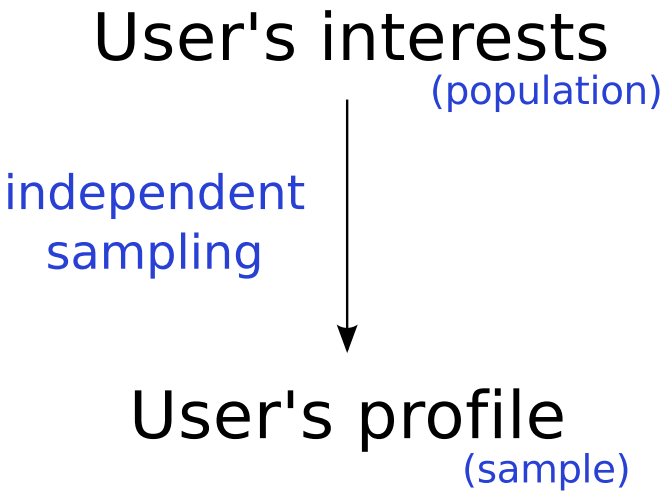
	wikipedia.com	gnu.org	debian.org	archlinux.org
Alice's profile	0	1	0	1
Bob's profile	1	0	1	1

Differential Privacy for Items

User's interests
(population)

- To have any utility at all, some information has to leak about the users interests

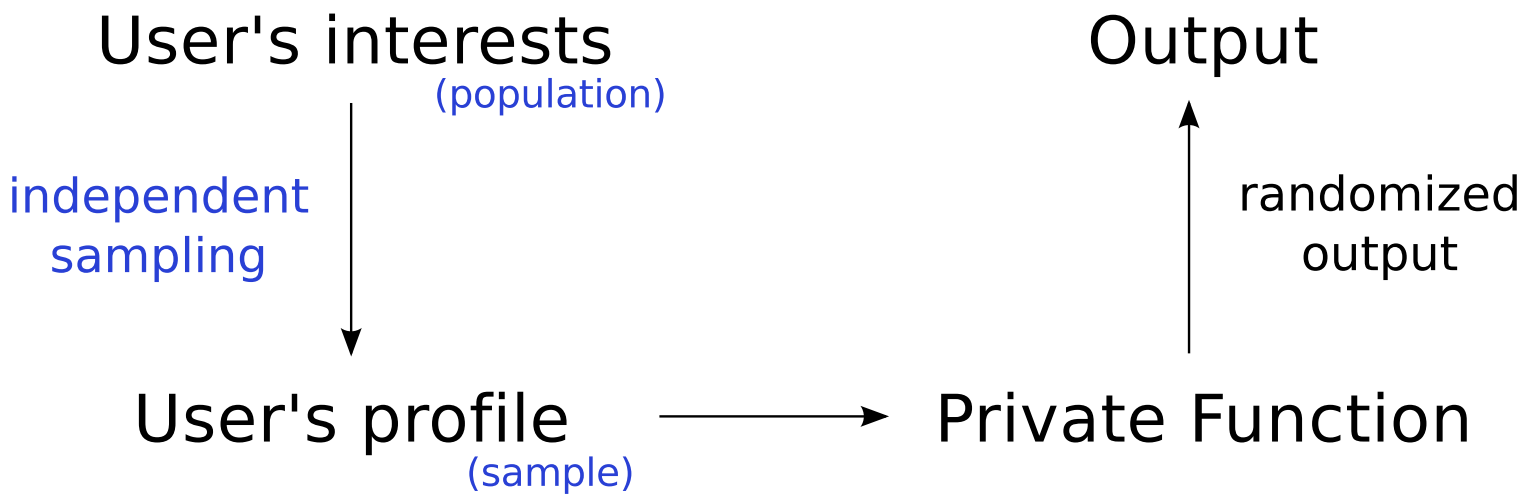
Differential Privacy for Items



- The user only has a sample of his interests (his profile)
- We assume a profile is sampled independently
  - Otherwise correlations may violate privacy [KM11]
  - Some solution (not part of this work): preprocess the profiles in a DP manner to remove correlations, for instance, by using dimensionality reduction or SVD

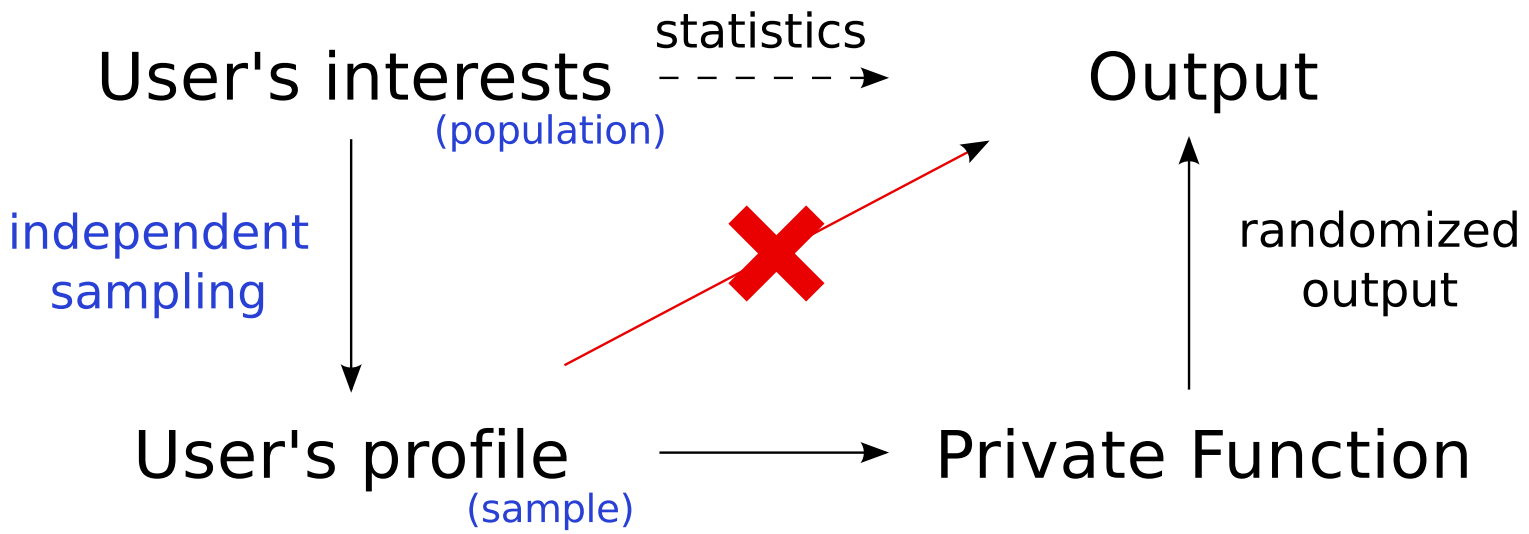
[KM11] Kifer and Machanavajjhala. No free lunch in data privacy. SIGMOD'11

Differential Privacy for Items



- A function of the profile, satisfying differential privacy, is released

Differential Privacy for Items



- Learning aggregate information about the user's interests (such as the similarity to another user) is not considered a privacy violation
- Inferring items of the profile is considered a privacy violation (evidence of participation of a particular item)
  - Obstruct linkage/deanonymization/reconstruction attacks
  - Plausible deniability
  - Decreases incentives to *not* add *an* item to the profile

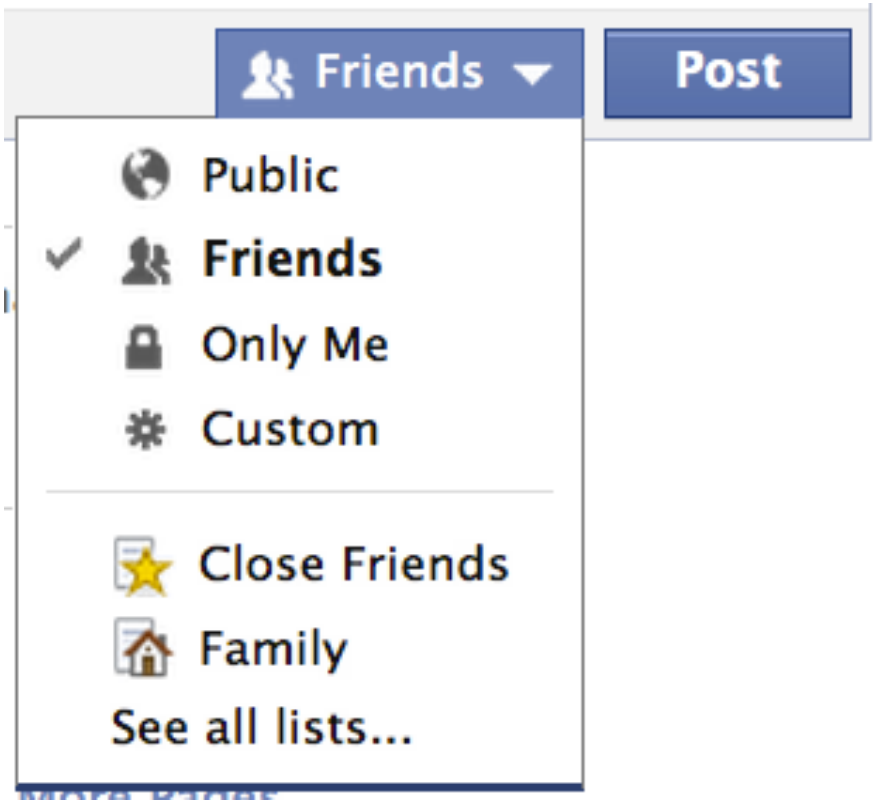
Traditional Solution

- Using the Laplacian mechanism for a function f with global sensitivity Δf
- Release

$$f(\mathbf{d}) + \mathsf{Lap}(\Delta f/\varepsilon) \ . \tag{1}$$

- Problem: assumes same privacy budget (ε) for all users and all items

Heterogeneity



Different people have different privacy expectations and some items may be considered more private than others

Heterogeneous Differential Privacy

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Definition ($(\varepsilon, v_1, \ldots, v_n)$-Differential Privacy (HDP))

A randomized algorithm $\mathcal{A}$ satisfies $(\varepsilon, v_1, \ldots, v_n)$-differential privacy if for all items i and for all neighboring profiles $\mathbf{d}, \mathbf{d}^{(i)}$, and $S \subset \mathrm{Range}(\mathcal{A})$:

$$\Pr[\mathcal{A}(\mathbf{d}) \in S] \leqslant \exp(\varepsilon v_i) \Pr[\mathcal{A}(\mathbf{d}^{(i)}) \in S] \ , \tag{2}$$

in which $\mathbf{d}$ and $\mathbf{d}^{(i)}$ differ on *at most* item i, and each privacy weight v_i is in the range $[0, 1]$, where 0 means absolute privacy, and 1 means traditional ε-differential privacy.

- HDP is composable

Modular Global Sensitivity

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Definition (Modular Global Sensitivity)

Modular Global sensitivity of a function f with respect to i is

$$\Delta_i f = \max_{\mathbf{d} \sim \mathbf{d}^{(i)}} \left| f(\mathbf{d}) - f(\mathbf{d}^{(i)}) \right| \quad . \tag{3}$$

The traditional global sensitivity is then

$$\Delta f = \max_i \Delta_i f \quad . \tag{4}$$

Key Observation

Let $\lambda = \Delta f/\varepsilon$ and $N = \text{Lap}(\lambda)$. From the PDF of the Laplacian distribution:

$$\frac{\Pr[f(\mathbf{d}) + N \in S]}{\Pr[f(\mathbf{d}^{(i)}) + N \in S]} \leqslant \exp(\Delta_i(f)/\lambda) \tag{5}$$

$$= \exp(\underbrace{c_i \overbrace{\Delta(f)/\lambda}^{\varepsilon}}_{\varepsilon_i}) \tag{6}$$

$$\leqslant \exp(\varepsilon) \ , \tag{7}$$

for some $0 \leqslant c_i \leqslant 1$.

Core Idea

Manipulating Δ_i controls ε_i:
The lower Δ_i is, the more private the item i

Satisfying HDP

- For a privacy vector $\mathbf{v}$, any $(\min_i v_i)$-differentially private mechanism satisfies HDP
- Sacrifices a lot of utility:
  - What if $\min_i v_i$ is much lower than other weights, or zero?
- Would like to use heterogeneity to achieve better utility

Stretching Mechanism

Given a profile $\mathbf{d}$ and privacy weights $\mathbf{v} \in [0,1]^n$, find a weight vector $\mathbf{w} \in [0,1]^n$ such that the modular global sensitivities for all i of

$$\hat{f} = f(w_1 d_1, \ldots, w_n d_n) \tag{8}$$

satisfy

$$\Delta_i(\hat{f}) \leqslant v_i \Delta f \tag{9}$$

then invoke the standard Laplacian mechanism $\hat{f} + N$

Theorem

The stretching mechanism achieves $(\varepsilon, v_1, \ldots, v_n)$-differential privacy

Impact of Stretching

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Theorem (Distortion)

$$\left| f(\mathbf{d}) - \hat{f}(\mathbf{d}) \right| \leqslant (1 - m) \left\| \nabla f(B \cdot \mathbf{d}) \right\| \left\| \mathbf{d} \right\| \quad , \tag{10}$$

in which

- $m = \min_i v_i$
- $B = cI + (1 - c)T$ *for some* $0 \leqslant c \leqslant 1$
- $T = \mathrm{diag}(\mathbf{w})$

Distortion for Inner Product

Distortion for the Inner Product

- Additive distortion (for inner product) is $\widetilde{O}(\sqrt{n})$ when the privacy weights are $1 - \widetilde{O}(1/n^{3/2})$
- The privacy weights should approach 1 as n grows, otherwise the distortion will grow beyond the desired bound
- This distortion does not hurt the utility since the lower bound on the noise for the two-party inner product is $\widetilde{\Omega}(\sqrt{n})$ [MMPRTV][2]

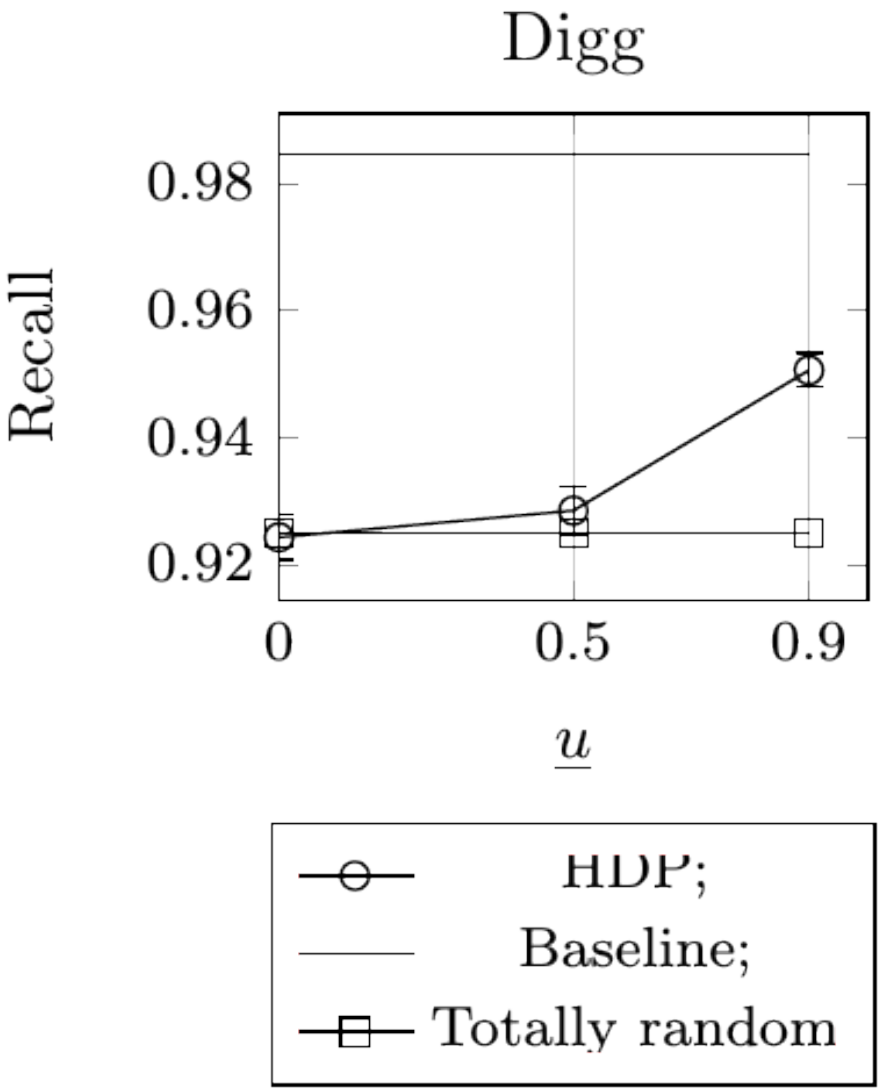
[2][MMPRTV] McGregor, Mironov, Pitassi, Reingold, Talwar, and Vadhan. The limits of two-party differential privacy. FOCS'10

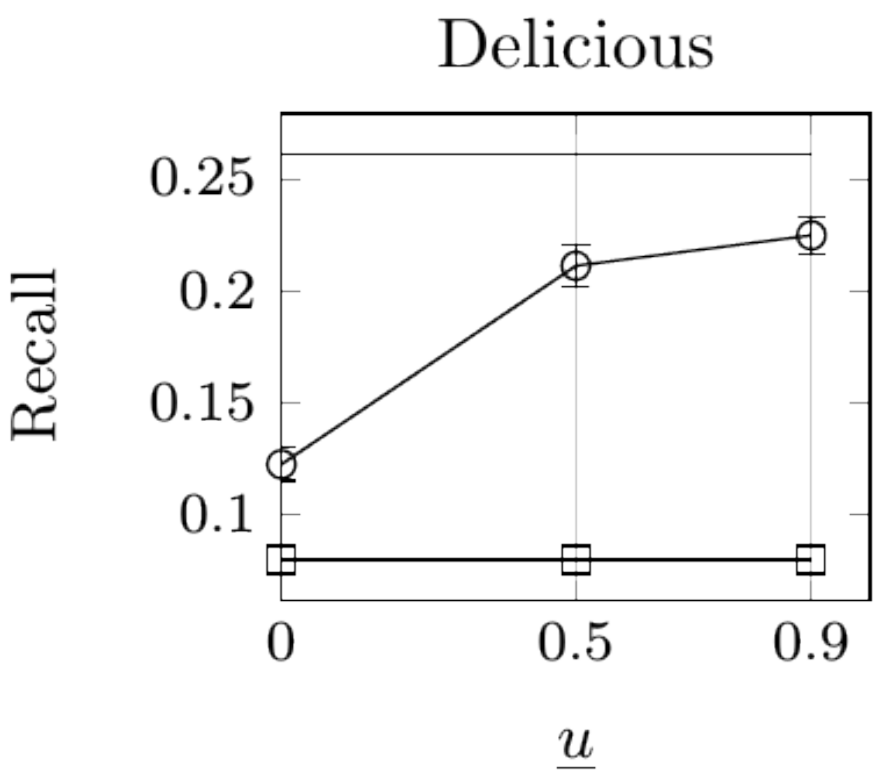
Privacy of $\mathbf{v}$

- The privacy weights themselves should be private
- If user A gives a high privacy weight to item j, it indicates that this item as a special interest for him
- Our stretching mechanism guarantees that the privacy weights themselves are protected with the standard ε-differential privacy: for all profiles $\mathbf{d}$ and neighboring privacy vectors $\mathbf{v}, \mathbf{v}'$

$$\max_{\mathbf{v} \sim \mathbf{v}'} |\hat{f}_{\mathbf{v}}(\mathbf{d}) - \hat{f}_{\mathbf{v}'}(\mathbf{d})| \leqslant \Delta f \tag{11}$$

Experimental Results





Related Work

JYC15 [3]

- Parallel line of work, with the following differences:
- There is one privacy weight per user
- Privacy weights are public
  - Can be determined only based on public data; like the user's occupation
  - Cannot be determined by the user's preference if his preference reflects private information

ESS15 [4]

- Previous Talk!

PGM14 [5]

- Uses rescaling as a mathematical tool to make query transformations (like Join) stable
  - To avoid worst-case sensitivity, for the purpose of better utility
- Incomparable to our work: Unclear how it could be applied to ensure different levels of privacy

[3] [JYC] Jorgensen, Yu and Cormode. ICDE'15
[4] [ESS15] Ebadi, Sands and Schneider. POPL'15
[5] [PGM14] Proserpio, Goldberg and McSherry. PVLDB 7(8): 637–648 (2014)

Conclusion

Summary

- Proposed a mechanism to ensure heterogeneous differential privacy
- Demonstrated that it protects both the items and the privacy weights
- Some functions do not lend themselves to heterogeneous differential privacy
  - The ℓ_0 function (the support of the input vector)
  - The minimum function (semantics of the function not preserved)

Future Work

- Investigate solutions for profiles with correlated items
  - Perhaps through dimensionality reduction
- Propose mechanisms with less distortions
  - Investigate pre/post processing techniques that can reduce distortion
    - May depend on the weight vector; must take care
- Find lower bounds on distortion for families of functions
- Investigate non-interactive mechanisms for HDP

Thank you!